Linear algebra -Midterm 1

1.

(a) Find $\begin{bmatrix} -1 & 0 \\ 2 & -3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 3 & -3 \end{bmatrix}$.

Solution: $\begin{bmatrix} 0 & -2 \\ 2 & -2 \\ 7 & -2 \end{bmatrix}$.

(b) Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$.

Solution: $\begin{bmatrix} 1 & -1 & 1 \\ -2 & 3 & -2 \\ 0 & -1 & 1 \end{bmatrix}$.

2. Let $A = \begin{bmatrix} 1 & 2 & 1 & -1 & 0 \\ 2 & 4 & 1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ -1 & -2 & 0 & 0 & 3 \end{bmatrix}$, and suppose that if we row reduce A we get the matrix $\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for the column space of A.

Solution: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix} \right\}$.

(b) Find the rank of A.
Solution: The rank is 3.

(c) Find a basis for the row space of A.
Solution: $[1, 2, 0, 0, 0], [0, 0, 1, -1, 0], [0, 0, 0, 0, 1]$.

(d) Find a basis for the nullspace of A.

Solution: $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

3. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation so that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

(a) Find the standard matrix representation of T.

Solution: $\begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}$.

(b) Find a basis for the kernel of T.

Solution: $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$.

(c) Find a basis for the range of T.

Solution: $\left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$.

4.

(a) Is $\{[x, y, z] \quad : \quad x + y + z = 1\}$ a subspace of $\mathbf{R}^3$? Give a reason for your answer.
Solution: It is not a subspace, since $[1, 0, 0]$ is in the space, but $2[1, 0, 0] = [2, 0, 0]$ is not, so it is not closed under scalar multiplications.

(b) Find the dimension of the subspace $\{[x + y, z, 0] \quad : \quad x, y, z \in \mathbf{R}\}$.
Solution: The dimension is 2.

(c) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be rotation about the origin by 90 degrees, *clockwise*. Find the standard matrix representation of T.
Solution: $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) Is the function $T : \mathbf{R}^2 \to \mathbf{R}^2$ given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 x_2 \\ x_2 \end{bmatrix}$$

a linear transformation? Give a reason for your answer.
Solution: It is not a linear transformation. $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, but $T\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, so

$$T\left(2\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) \neq 2T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right).$$